2. FUNCTIONS

Evaluating a Function

FUNCTIONS TERMINOLOGY

Relationships

- *A relation is any formula or rule that connects 2 sets of numbers.*

  i.e. $y = 2x + 3$

- *Domain:* all the numbers that exist for x

- *Range:* all the numbers that exist for y

Domain, Range and Functions

Finding the Domain and Range of Graphs

Functions

- A function is a relationship where for every x value, there is only ONE y value

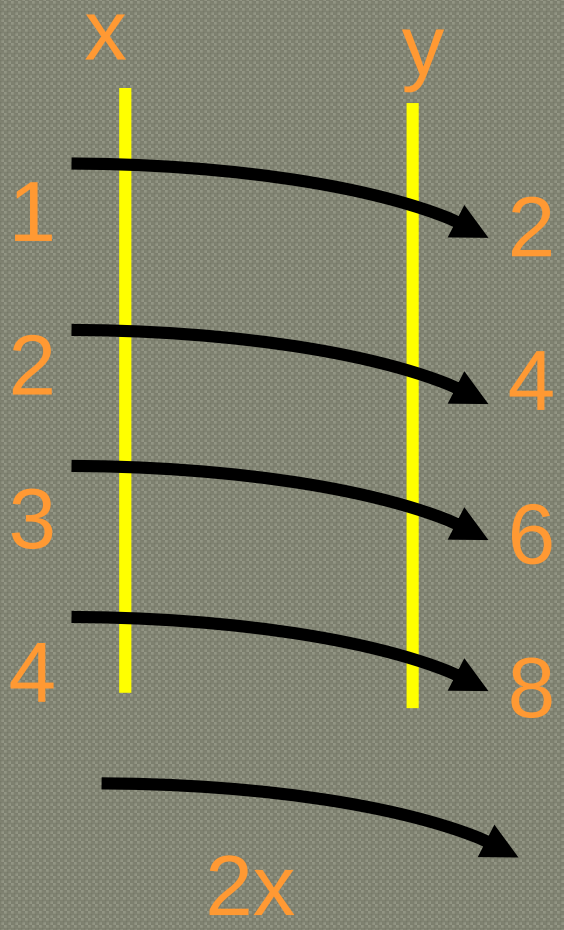
OR

A function is defined as a one-to-one relationship

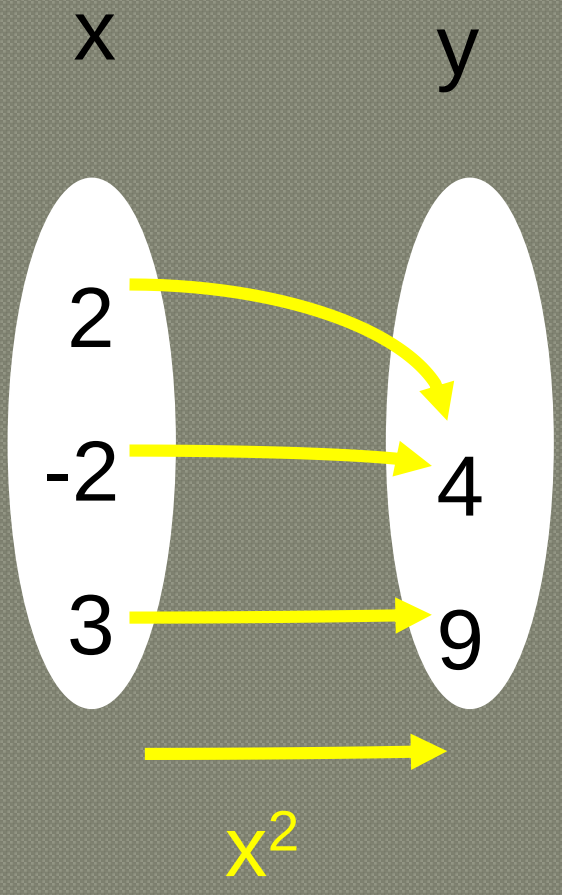
- There are 2 ways to establish if a relationship is a function
  i) Mapping
  ii) Vertical line test

i) Mapping

- *One-to-one mapping:* ✓ *Is a function*

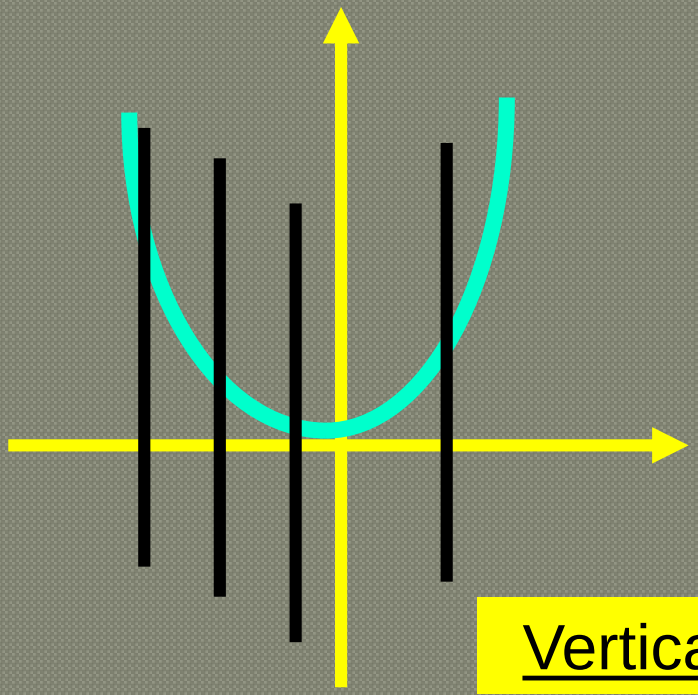


- *Many-to-one mapping:* × *Is NOT a function*

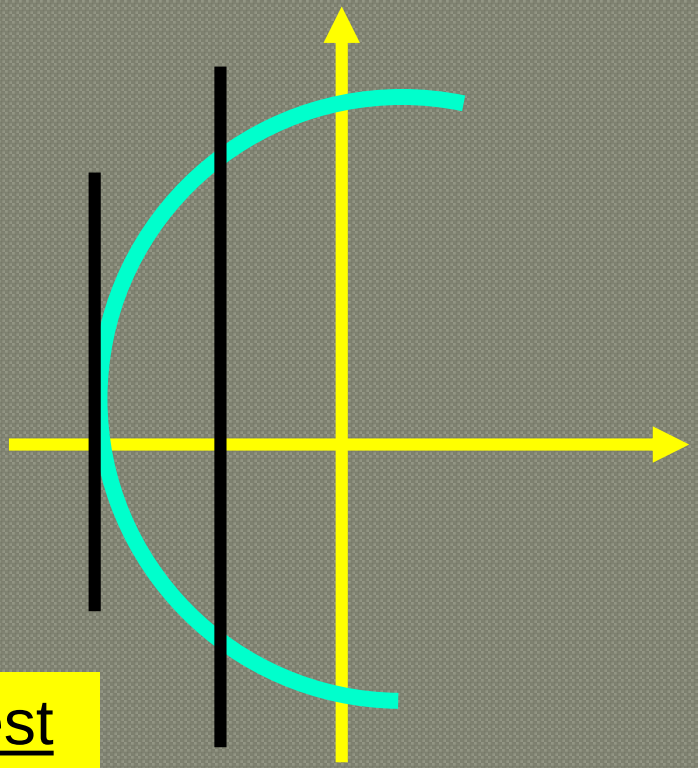


ii) Vertical Line Test

- Vertical line only cuts the graph in one place:
  - ✓ Is a function



- Vertical line cuts the graph in more than one place:
  - × Is NOT a function



Vertical Line Test

INVERSE FUNCTIONS

Inverse Functions

- *A function g(x) is the inverse of function f(x) if …*

the domain of g(x) is the range of f(x) and

the range of g(x) is the domain of f(x).

i.e. the x and y values are interchanged

Introduction to Inverse Functions

Inverse Function Graphs

Example: Inverse of a Straight Line

- $f(x)$: $y = 2x + 3$

y-int: $(0; 3)$

x-int: $(\frac{-3}{2}; 0)$

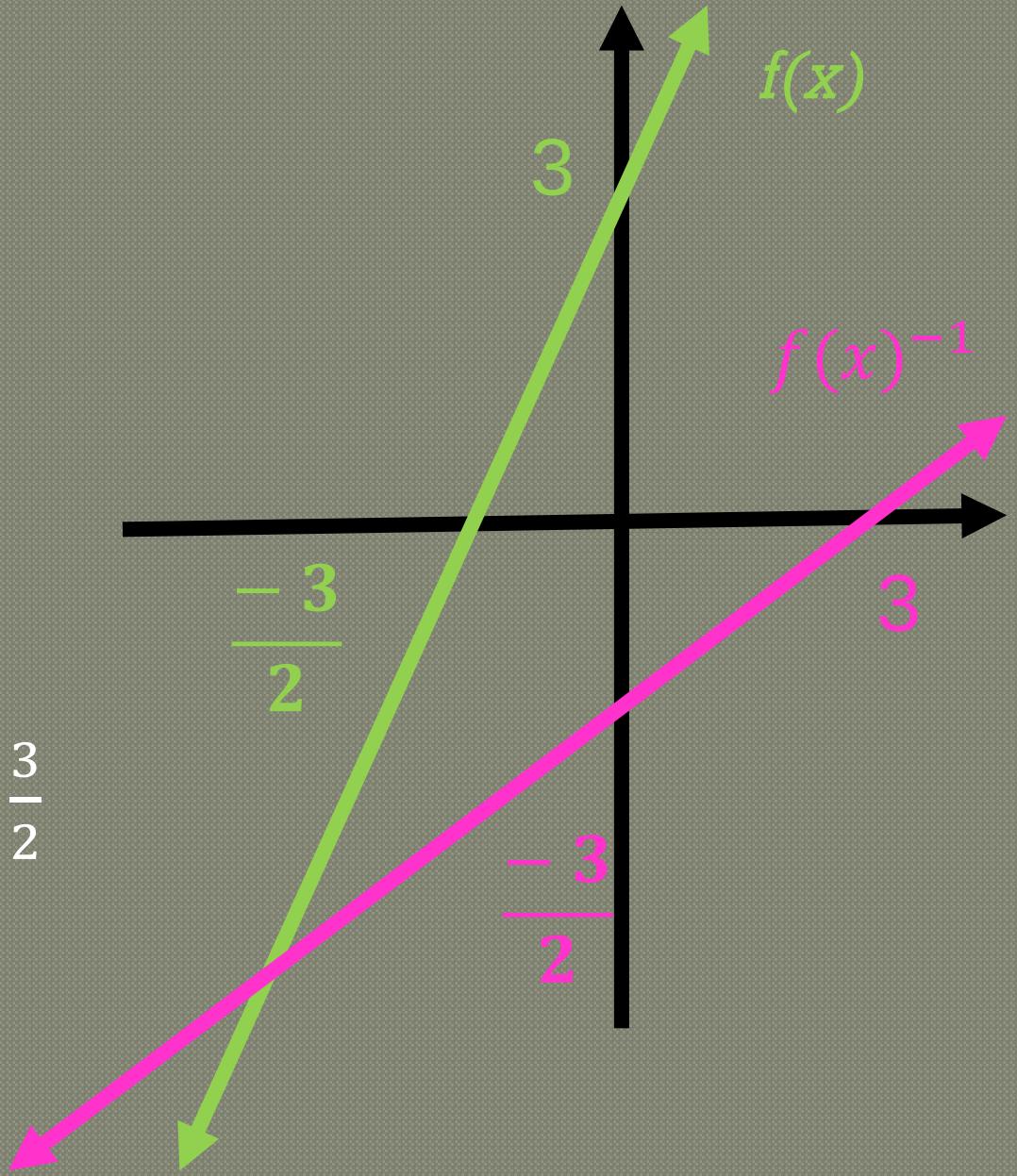
$f(x)^{-1}$: $x = 2y + 3$

$-2y = -x + 3$

$y = \frac{1}{2}x - \frac{3}{2}$

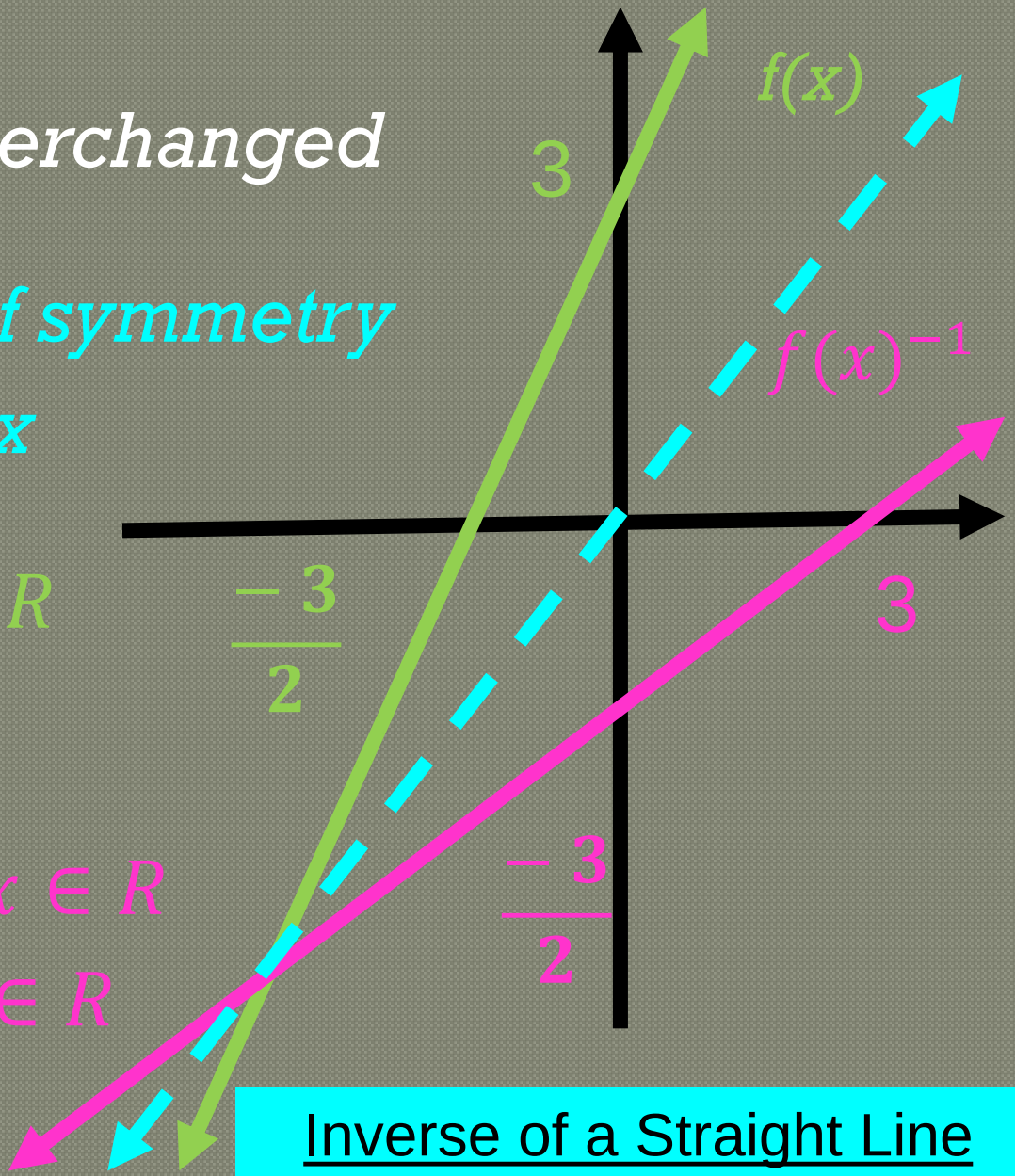
y-int: $(0; \frac{-3}{2})$

x-int: $(3; 0)$



Example: Inverse of a Straight Line

- *x and y values interchanged*
- *∴ there is a line of symmetry about the line y=x*
- *Domain f(x): $x \in R$*
  Range f(x): $y \in R$
- *Domain $f(x)^{-1}$: $x \in R$*
  Range $f(x)^{-1}$: $y \in R$



Inverse of a Straight Line

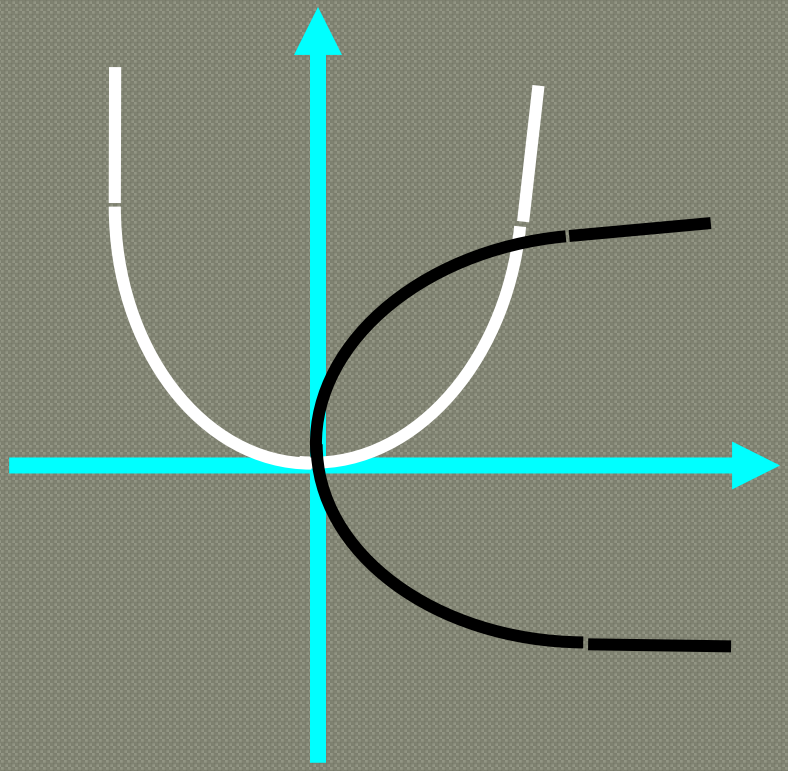
Example: Inverse of a Parabola

$y = 2x^2$

$y = 2x^2$ is a function

$x = 2y^2$

$y = \pm\sqrt{\frac{x}{2}}$



$y = \pm\sqrt{\frac{x}{2}}$ is NOT a function (fails the vertical line test)

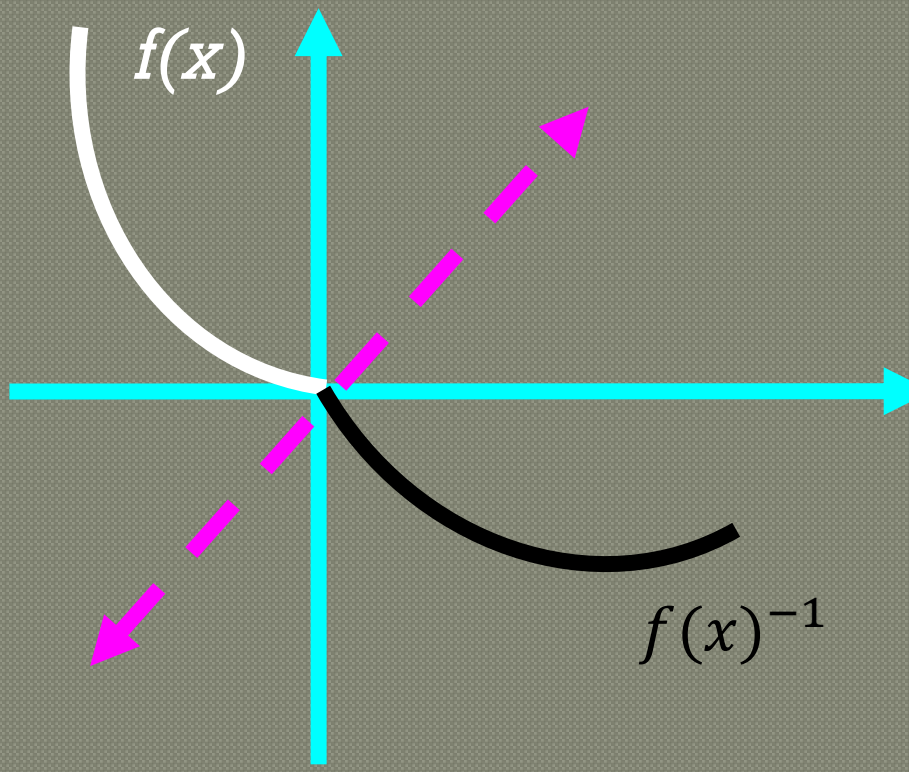
We can put restrictions on the original parabola, so that the inverse will be a function.

Example: Inverse of a Parabola

- *Sketch:* $f(x) = x^2 \; ; \; x \leq 0$
- Inverse: $f(x)^{-1} = -\sqrt{\frac{x}{2}}$
- *Line of symmetry: y=x*
- *Domain f(x):* $x \leq 0$

  Range f(x): $y \geq 0$
- *Domain* $f(x)^{-1}$ *:* $x \geq 0$

  Range $f(x)^{-1}$ *:* $y \leq 0$

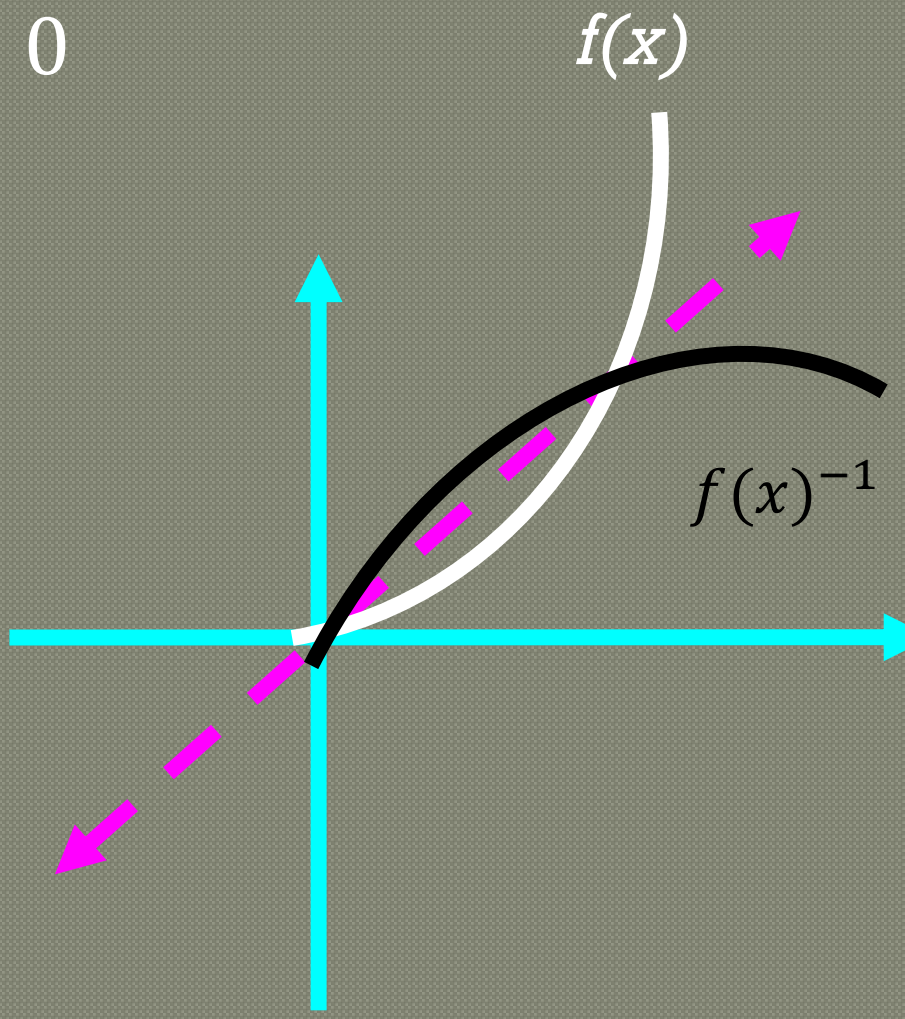


Example: Inverse of a Parabola

- *Sketch:* $f(x) = x^2 \; ; \; x \geq 0$
- Inverse: $f(x)^{-1} = \sqrt{\frac{x}{2}}$
- *Line of symmetry: y=x*
- *Domain f(x):* $x \geq 0$

  Range f(x): $y \geq 0$
- *Domain* $f(x)^{-1}$ *:* $x \geq 0$

  Range $f(x)^{-1}$ *:* $y \geq 0$



Inverse of a Parabola

EXPONENTIAL & LOG GRAPHS

Recap! Exponential Laws

- $a^0 = 1$
- $a^n . am = a^{m+n}$
- $(a^m)^n = a^{mn}$
- $(ab)^n = an . b^n$
- $a^n \div a^m = a^{n-m}$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

This should be "second-knowledge"!

Recap! Exponential Equations

Solve for x:

1. $5^x = 625$ — *Write 625 in terms of base 5*

   $5^x = 5^4$ — *Since bases are the same …*

   $x = 4$ — *We equate the exponents*

2. $5^x = 30$ — *30 CANNOT be written in term of base 5!*

   ?? Now what ??

   We use LOGS …

Definition of a Log

- The inverse of an exponent is a log

Exponential equation:

$x = a^y$

Interchange x and y for inverse:

$y = a^x$

Log equation:

$y = log_a x$ where $a, y > 0$ and $a \neq 1$

Interesting! The Log Scale

Examples: Converting between exponential and log forms

Write in log form:

1. $y = 5^x$

   $x = 5^y$

   $y = log_5 x$

2. $y = 3^{-x}$

   $y = \left(\frac{1}{3}\right)^x$

   $x = \left(\frac{1}{3}\right)^y$

   $y = log_{\frac{1}{3}} x$

- *First interchange x and y*
- *The base of the exponent becomes the base of the log*

Converting Between Exponential & Log Form

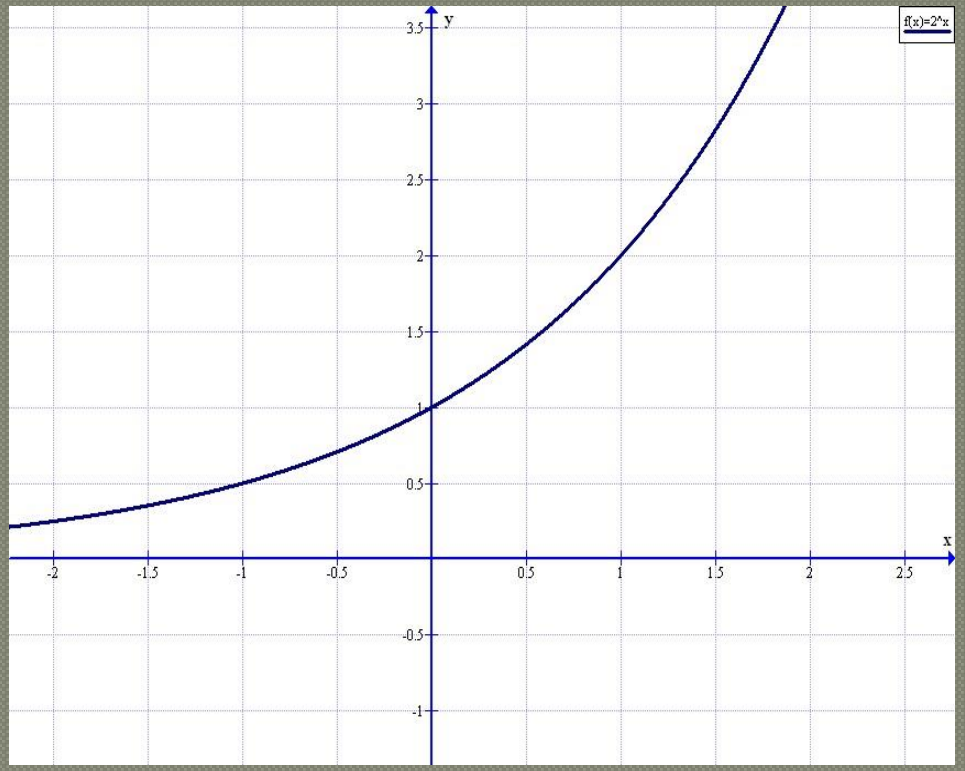
Solving Basic Log Equations

The Inverse Graphs of Exponents & Logs

Sketching Exponential and Log Graphs

Increasing exponential graph: $f(x)$: $y = 2^x$

- *No x-intercept as asymptote at $y = 0$*
- *y-int:* $y = 2^0 = 1$
- *Domain:* $x \in R$
- *Range:* $y > 0$



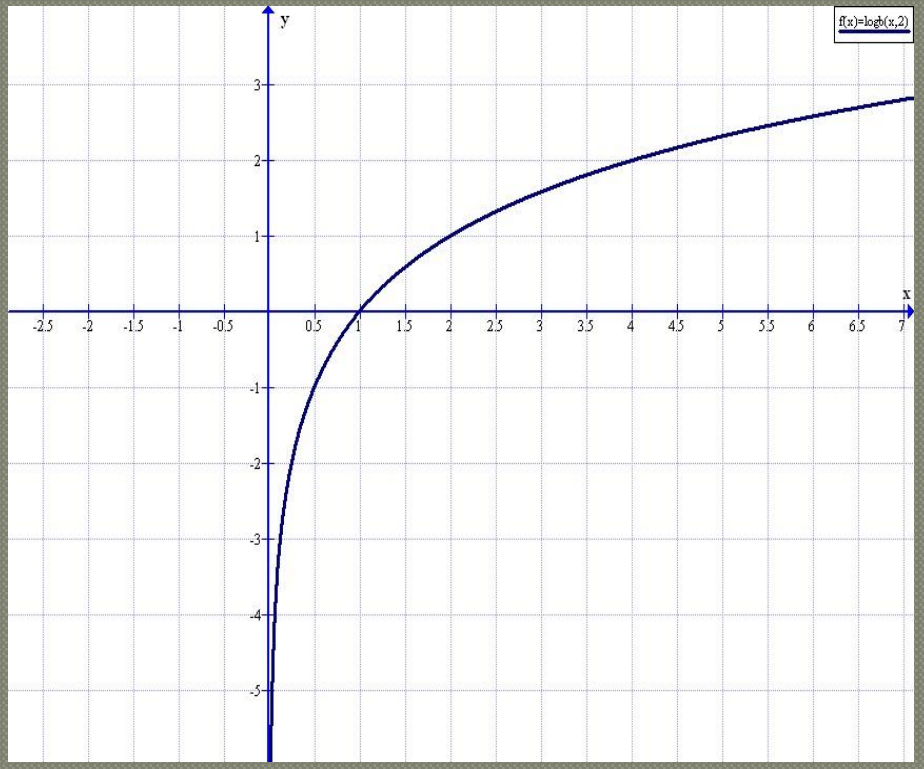
Sketching Basic Exponential Graphs

Sketching Exponential and Log Graphs

Increasing log graph: $f(x)$: $y = log_2 x$

- *No y-intercept as asymptote at $x = 0$*
- ***x-int:*** $0 = log_2 x$

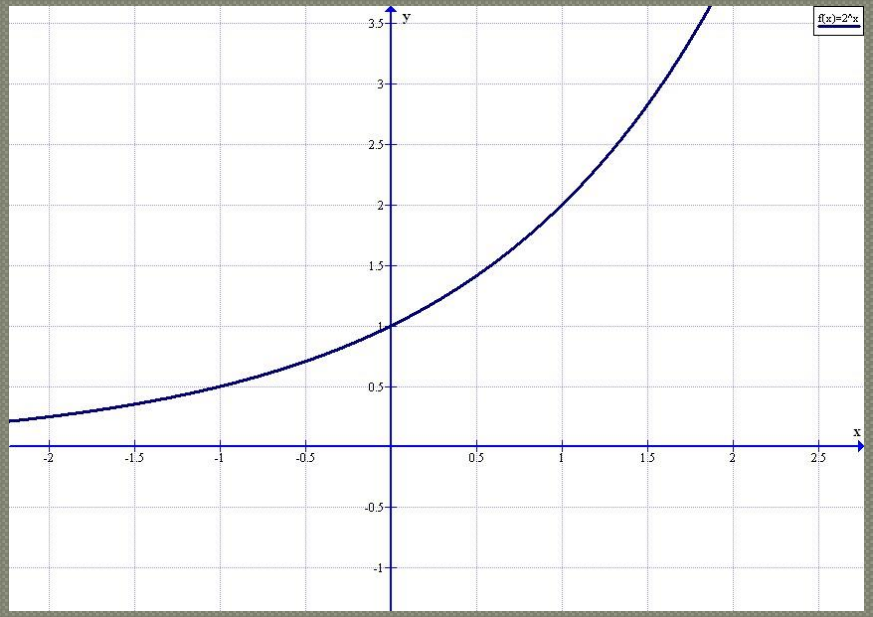
  $x = 2^0 = 1$
- *Domain:* $x > 0$
- *Range:* $y \in R$

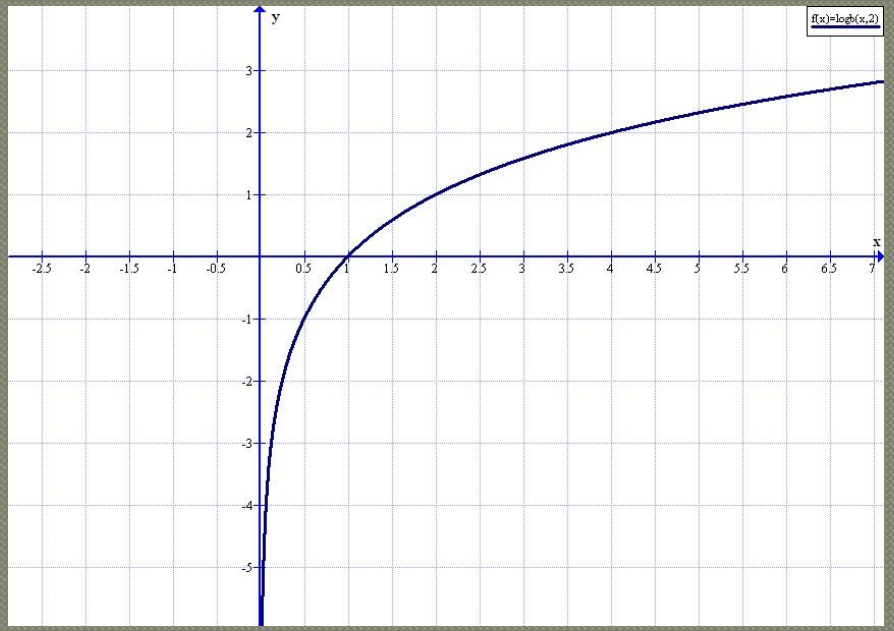


Sketching Basic Log Graphs

Increasing Exponential
Graph: $y = 2^x$

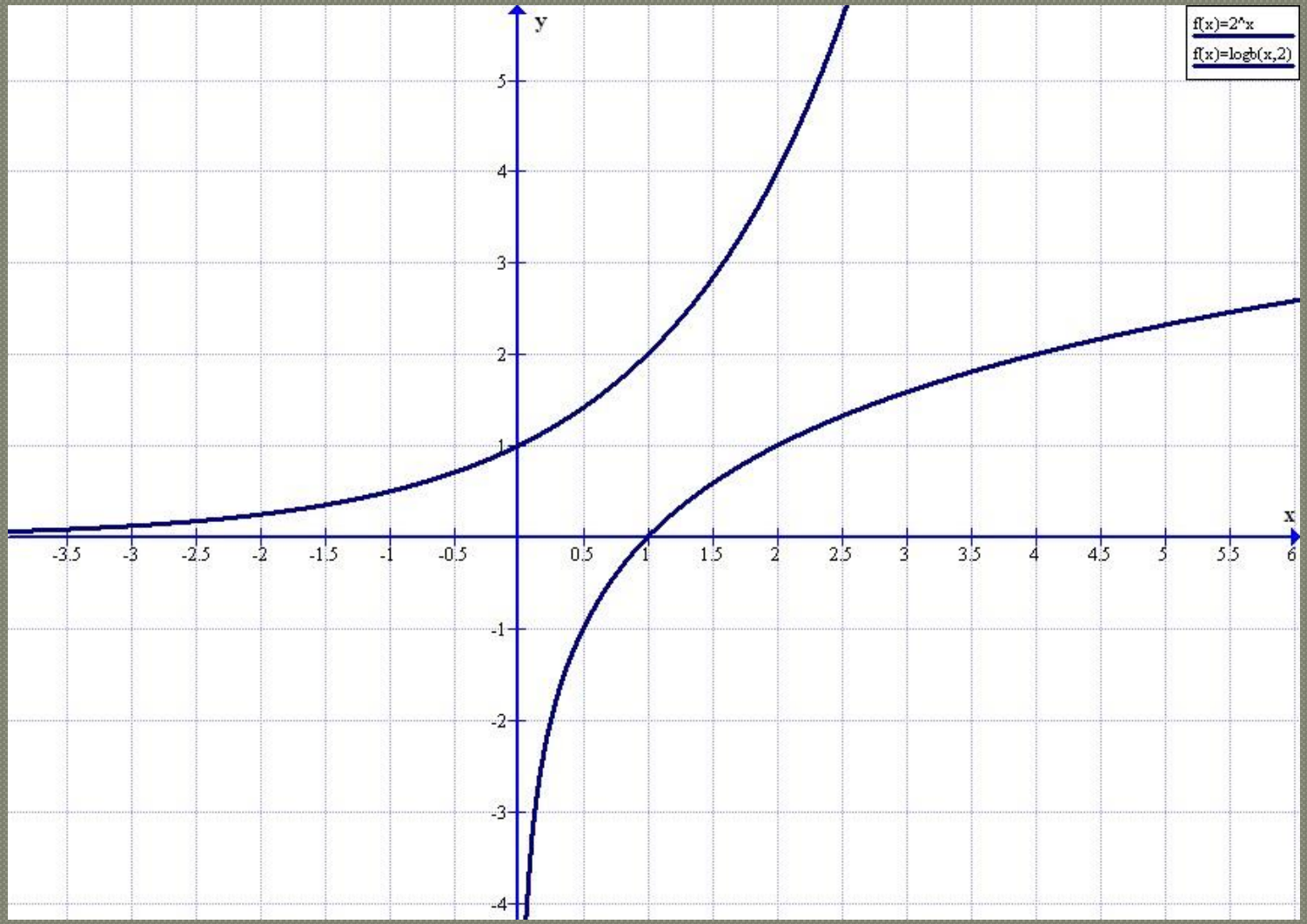
Increasing Log
Graph: $y = \log_2 x$





Together ….

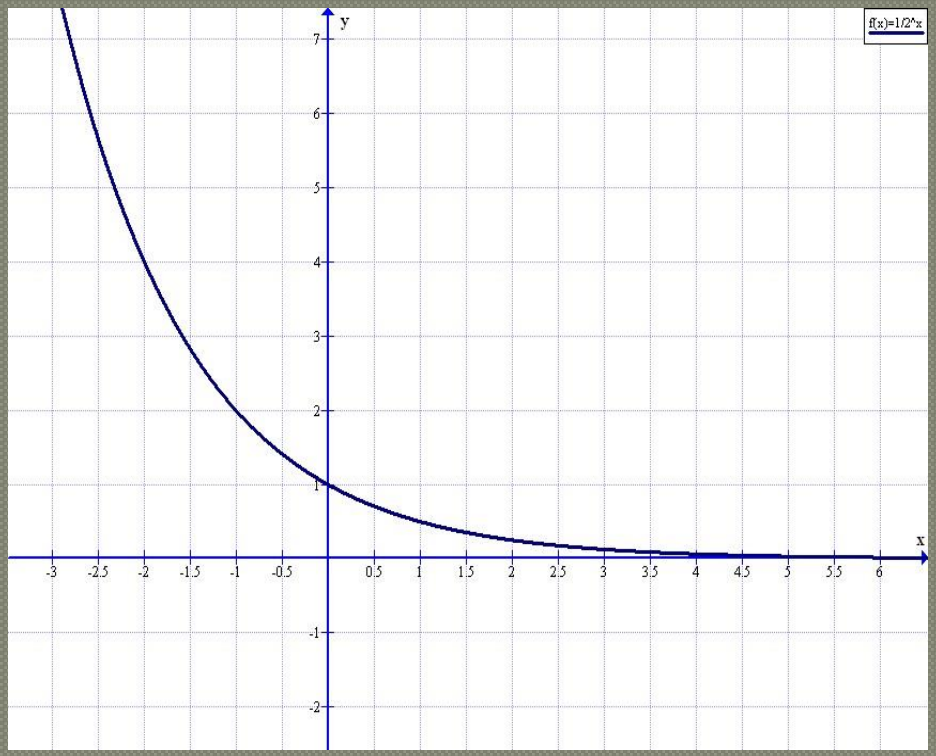
Increasing Exponential and Log Graphs ….



Sketching Exponential and Log Graphs

Decreasing exponential graph: $f(x)$: $y = \left(\frac{1}{2}\right)^x$

- *No x-intercept as asymptote at y = 0*
- *y-int:* $y = \left(\frac{1}{2}\right)^0 = 1$
- *Domain:* $x \in R$
- *Range:* $y > 0$



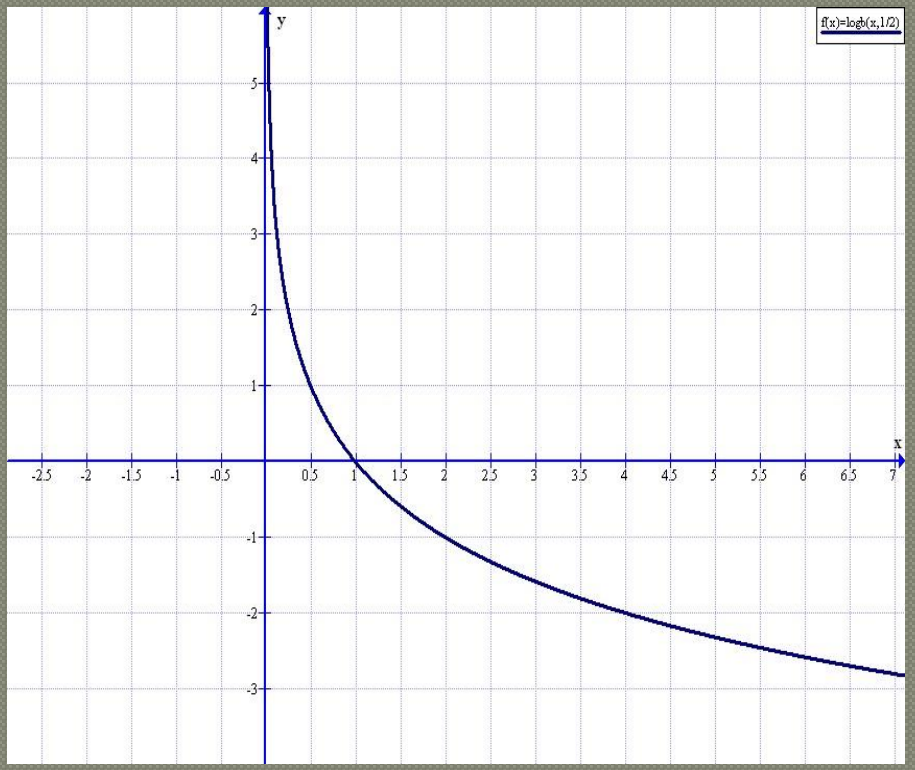
Sketching Exponential and Log Graphs

Decreasing log graph: $f(x)$: $y = log_{\frac{1}{2}} x$

- *No y-intercept as asymptote at x = 0*
- *x-int:* $0 = log_{\frac{1}{2}} x$

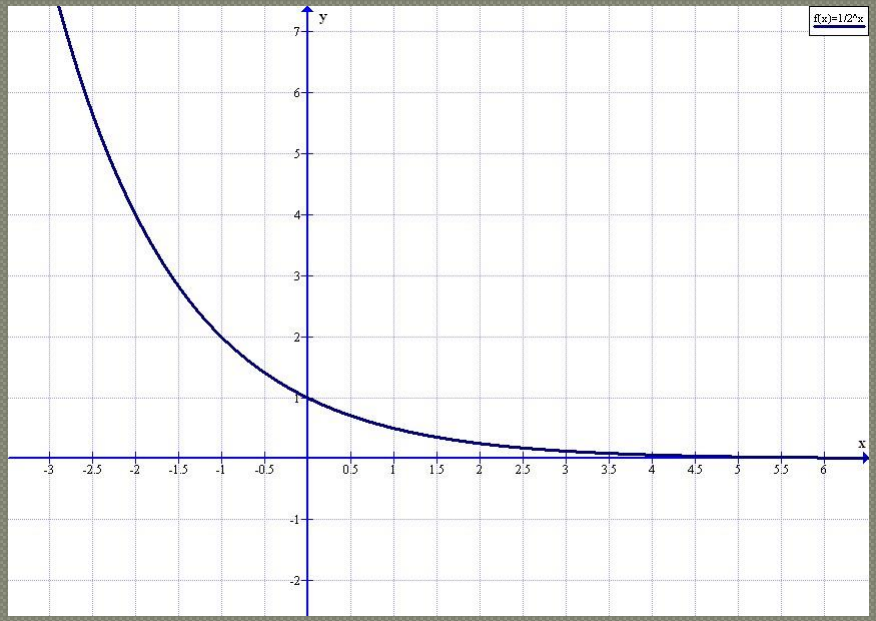
$$x = \left(\frac{1}{2}\right)^0 = 1$$

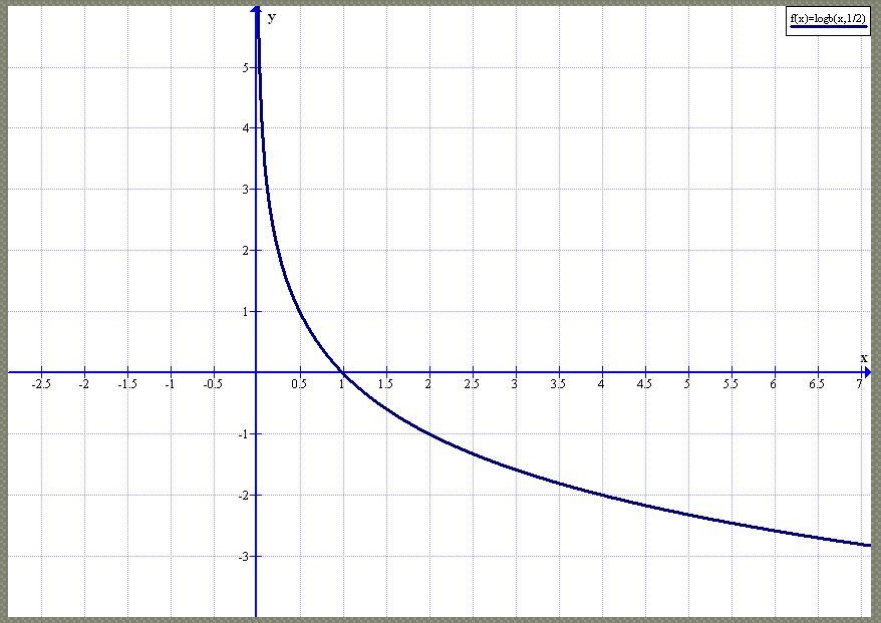
- *Domain:* $x > 0$
- *Range:* $y \in R$



Decreasing Exponential Graph: $y = \frac{1}{2}^x$

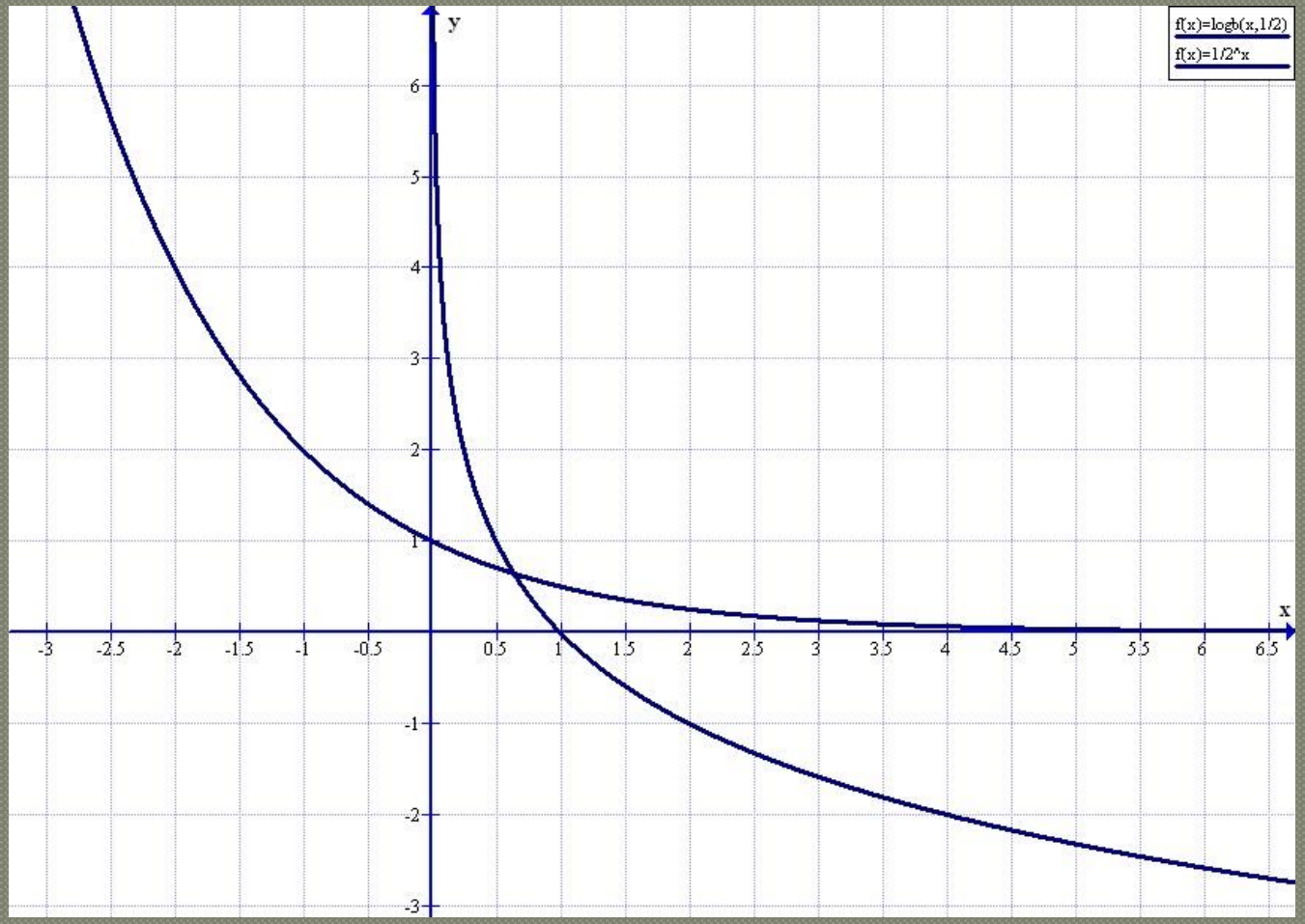
Decreasing Log Graph: $y = \log_{\frac{1}{2}} x$





Together ….

Decreasing Exponential and Log Graphs ….



Exponential and Log Graphs

SOLVING CUBIC EXPRESSIONS

Remainder Theorem

- If $f(x)$ is divided by $x - a$, then the remainder = $f(a)$

- Examples:

  i) Determine the remainder of $f(x) = 2x^3 - 4x^2 + x + 2$ when divided by $x + 1$.

$$f(-1) = 2(-1)^3 - 4(-1)^2 + (-1) + 2 = -5$$

Finding the Remainder

Remainder Theorem

- *Examples:*

  ii) *Determine the value of m if the remainder of* $f(x) = x^3 - x^2 + m - 12$ *is 10 when divided by* $x - 2$.

$$10 = (2)^3 - (2)^2 + m - 12$$

$$m = 18$$

Complex Remainder Problem

Factor Theorem

- A factor is a number that divides exactly into another number, with NO remainders
- Examples:

  i) Factors of 12: 1; 2; 3; 4; 6; 12

  ii) Factors of $x^2 - x - 6$:

  $x^2 - x - 6 = 0$ [indicates no remainders)

  $(x - 3)(x + 2) = 0$

  $\therefore \ x = 3 \ \ and \ \ x = -2 \ \ are \ factors$

Understanding the Factor Theorem

Complex Factor Theorem Problem

Remainder & Factor Theorem

Visualizing Factors & Remainders

Calculating Remainders given Factors

Example: Finding Factors of Cubic Expressions

- $f(x) = x^3 + 3x^2 - x - 3$

If $(x- p)$ is a factor of $f(x) = ax^2 + bx + c$ then $f(p) = 0$

$$\text{Remainder} = f(1)$$
$$= 1^3 + 3(1)^2 - (1) - 3$$
$$= 0$$

Therefore $(x - 1)$ is a factor

- *Note! f(1) did not equal zero we would have to continue to find f(-1), f(2), f(3), f(-3) etc until the remainder equalled 0.*

Solving Cubic Expressions

1. Solve for x: $f(x) = x^3 - x^2 - 8x + 12$

$f(1) = (1)^3 - (1)^2 - 8(1) + 12 = 4$

$f(2) = (2)^3 - (2)^2 - 8(2) + 12 = 0$

$\therefore x - 2$ is a factor

$\therefore$ **x – 1 is NOT a factor**

$f(x) = x^3 - x^2 - 8x + 12$

$0 = (x - 2)(x^2 + 1x - 6)$

$0 = (x - 2)(x + 3)(x - 2)$

$\therefore \; x = 2 \text{ or } x = -3 \text{ or } x = 2$

$x \times ? = x^3$

$? = x^2$

$-2 \times ? = 12$

$? = -6$

$-2x^2 + ? = -x^2$

$? = 1x^2$

Solving Cubic Expressions

2. Solve for x: $f(x) = 2x^3 - 7x^2 + 2x + 3$

$f(1) = (1)^3 - (1)^2 - 8(1) + 12 = 4$

$\therefore x - 1$ is a factor

Solving Cubic Expressions

$f(x) = 2x^3 - 7x^2 + 2x + 3$

$0 = (x - 1)(2x^2 - 5x - 3)$

$0 = (x - 1)(2x + 1)(x - 3)$

$\therefore \; x = 1 \text{ or } x = -\frac{1}{2} \text{ or } x = 3$

$x \times ? = 2x^3$

$? = 2x^2$

$-1 \times ? = 3$

$? = -3$

$-2x^2 + ? = -7x^2$

$? = -5x^2$